

# Trellis Coded Modulation having Rate 7/8, 256 State with 256 QAM modulation designed for fading Channel

Raj Kumar Goswami

Gayatri Vidya Parishad College of Engineering for Women, Visakhapatnam, A.P

**Abstract:** - A Trellis Coded Modulation (TCM) scheme with a coding rate of 7/8, 256 states and utilizing 256-QAM modulation, specifically optimized for fading channels has been proposed in this paper. The proposed TCM scheme demonstrates significant performance improvements in challenging channel conditions, where fading due to multipath propagation and environmental factors typically degrades communication reliability. By introducing convolutional coding at a 7/8 rate, the TCM design effectively enhances the robustness of data transmission, making the system more resilient to fading effects. A critical outcome of this study is the demonstrated coding gain, where the TCM scheme beats uncoded 256-QAM by almost 13 dB while compared with respect to Signal-to-Noise Ratio (SNR). This substantial gain illustrates the system's enhanced ability to maintain low Bit Error Rates (BER) in the presence of fading. Performance evaluations using the Viterbi algorithm for decoding confirm the scheme's effectiveness, particularly in high-SNR regions. The analysis underscores the advantages of combining high-state TCM with high-order QAM modulation to achieve both high data rates and reliability in fading channels. This design proves highly suitable for applications where consistent performance is required under varying and challenging channel conditions.

**Keywords:** Trellis Coded Modulation (TCM), Error Correction Coding, 256-State Encoder, Fading Channels, Coding Gain, Bandwidth Efficiency

## I. INTRODUCTION

TCM is powerful scheme incorporating both modulation and error correction in a unified scheme, primarily used to enhance data transmission reliability and efficiency in digital communication systems. Introduced by Gottfried Ungerboeck in the 1980s, TCM integrates convolutional coding with modulation to achieve robust performance without increasing bandwidth. This technique is especially useful in bandwidth-limited environments, as it offers coding gain without the need to expand the signal's bandwidth.

Unlike conventional error correction methods, which typically require additional redundant bits, TCM merges coding with modulation, thereby maintaining the same symbol rate while achieving better error performance. In TCM, higher-dimensional signal constellations, such as Quadrature Amplitude Modulation (QAM), are utilized. By carefully mapping symbols in the signal constellation through convolutional encoding, TCM spreads the transmitted symbols for maximizing minimum Euclidean distance amongst them. This increased distance makes it more challenging for noise or interference to cause symbol errors, thus lowering the Bit Error Rate (BER).

A hallmark of TCM is constellation expansion. For example, if a system previously used 4-QAM, a TCM implementation might shift to an 8-QAM or 16-QAM constellation. However, not all additional points in the expanded constellation are used for data transmission. Instead, they are selected based on specific rules to maintain reliable data. This selection process, known as set partitioning, ensures that coded symbols are mapped to constellation points in a way that enhances error protection by grouping symbols into subsets with similar distance properties.

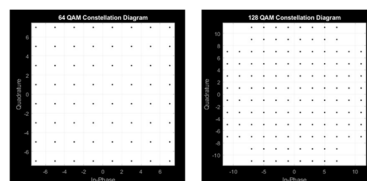


Fig. 1 Increasing size of signal set reduces spacing between individual points

The term "trellis" in TCM originates from the trellis diagram representation of convolutional codes, where states and transitions illustrate how each symbol's transmission depends on previous symbols. The trellis structure allows TCM to use memory in encoding symbols, making the scheme resilient to isolated errors. At the receiver, the Viterbi algorithm is typically employed to perform maximum likelihood decoding, following the most probable path through the trellis. This process allows for optimal error correction based on the received signal sequence.

TCM is particularly advantageous in environments with multipath fading, such as wireless and satellite channels. In fading channels, signal amplitudes can vary due to environmental factors and multipath propagation, leading to signal degradation. The convolutional coding inherent in TCM improves resilience to these variations, as the increased Euclidean distance in the constellation helps differentiate between symbols even in fluctuating

signal conditions. The coding gain provided by TCM allows for performance improvements in Signal-to-Noise Ratio (SNR) without increasing power or bandwidth requirements.

With its robust error correction and bandwidth efficiency, TCM is widely used in applications that require reliable, high-rate data transmission. These include modems, satellite communications, mobile networks, and digital broadcasting systems. TCM's ability to operate efficiently in fading environments makes it suitable for mobile and wireless applications, where channel conditions can change rapidly. By optimizing TCM for different modulation schemes and channel conditions, high data rates and reliability can be achieved, which is most suitable for modern communication systems.

Therefore, Trellis Coded Modulation remains a foundational scheme in digital communications, providing a powerful tool to combat noise and fading without sacrificing spectral efficiency. Its integration of coding with modulation enables enhanced reliability, making TCM an ideal choice for systems where high data rates and robust performance in challenging channels are paramount.

Dividing the signal constellation allows for efficient encoding combined with modulation, resulting in stronger error correction and improved performance [4]. In Trellis Coded Modulation, the encoder's output determines the selection of signal set, while added systematic input specifies exact symbol inside that set. This structured approach supports both reliable data transmission and resilience against noise.

This dual process of selecting signal sets and mapping symbols creates effective encoding and modulation design that strengthens error correction along with enhancement of performance. By carefully coordinating these elements, Trellis Coded Modulation achieves high performance, making it well-suited for reliable data transmission in challenging communication environments.

In convolutional codes, one of the most critical types of errors is the pairwise error. This type of error happens when decoder incorrectly chooses an alternative sequence instead of one originally sent by transmitter. Such errors significantly impact the reliability of the decoding process, as they can lead to the propagation of inaccuracies in the decoded data, thereby undermining the overall effectiveness of the error correction.

Pairwise errors refer to instances when the receiver confuses one possible sequence of symbols for another, resulting in a decoding error. This phenomenon occurs because TCM uses a trellis structure where different sequences of symbols correspond to different paths through the trellis. Pairwise errors typically happen when two distinct paths have a small Euclidean distance between them, making it difficult for the receiver to distinguish between them, especially under noisy or fading channel conditions. The likelihood of pairwise errors depends heavily on factors such as SNR, minimum Euclidean distance between paths, and the channel's fading characteristics. In conditions where the SNR is low or fading is present, the probability of pairwise errors increases because these factors make it easier for the receiver to misinterpret one sequence for another.

Mitigating pairwise errors in TCM requires different strategies depending on the SNR conditions. In high SNR scenarios, noise has minimal impact on the signal, making pairwise errors less frequent. Under these conditions, TCM schemes are often enhanced to increase the minimum Euclidean distance among trellis trajectories, which enhances the system's ability to distinguish between paths accurately. Additionally, advanced decoding techniques like the Viterbi algorithm can leverage the high SNR to make more precise path decisions, thereby minimizing the chance of selecting the incorrect path.

In medium SNR environments, noise plays a more substantial role in affecting the signal, increasing the likelihood of pairwise errors. To address this, TCM employs set partitioning, a technique that groups constellation points in such a way that symbols with low Hamming distances are assigned to high-energy states. This approach helps to maintain a minimum Euclidean distance between possible paths, reducing the probability of errors even at moderate SNRs. Additional layers of error-correcting codes can also be integrated to mitigate errors resulting from closely spaced paths, further enhancing robustness.

Low SNR conditions pose a significant challenge due to the high noise level, which makes distinguishing between similar paths especially difficult. In these scenarios, TCM schemes often increase redundancy by using lower coding rates, such as  $2/3$  or  $3/4$ , which enhances minimum Euclidean distance between symbols and reduces the risk of pairwise errors. Adaptive modulation techniques can also be applied; for instance, switching from modulation of higher order (e.g., 128-QAM) to lower one (e.g., 32-QAM) reduces the number of close pairs, thus improving reliability. Soft-decision decoding techniques, which weigh possible paths based on their likelihood, further help in low SNR environments by enhancing the accuracy of decoding decisions.

Therefore, mitigating pairwise errors in TCM involves a combination of optimizing the trellis structure, using set partitioning, adjusting redundancy levels, and adapting modulation schemes according to the SNR. These strategies collectively improve the robustness of TCM, ensuring high-performance data transmission across various noise and fading conditions.

Trellis Coded Modulation (TCM), when coupled with appropriate interleaver, it provides enhanced coding gains compared to uncoded schemes, particularly in fading environments. However, following well-defined design criteria [6] during the code design procedure is essential to fully realize these benefits. Proper design ensures that the TCM scheme can effectively combat the effects of fading, thereby enhancing system reliability and performance.

Organization of paper is brought out subsequently. Brief outline of System Model has been provided in first section, detailing its key components and their functions. Next, Section II outlines the proposed design methodology along with the guidelines for developing an optimal code for a 7/8 rate, 256-State, 256-QAM Trellis Coded Modulation scheme, specifically tailored for fading channel conditions. This section offers an in-depth view of the design methodology for developing a code that aligns efficiently and effectively with the system parameters. Section III describes the code construction process in detail, outlining each step involved in its development. Section IV discusses the performance analysis of the proposed design, evaluating its effectiveness in fading channel conditions. In Section V, the results are presented along with an analysis of the insights gained from the simulation outcomes. This section brings out the comprehensive overview of the achieved results and their relevance to the proposed design and the overall system performance.

## II. SYSTEM MODEL

The end-to-end diagram of a Trellis Coded Modulation (TCM) system as shown in figure 2 begins with an Input Source in Binary Form and progresses through various stages designed to encode, modulate, transmit, decode, and finally compare the output with the input to assess error rates. Following is a step-by-step explanation of each block:

- a. **Input Source in Binary Form.** The process starts with a source that generates binary data, representing the original information to be transmitted. This data could come from any digital source, such as text, image, or audio data, converted into a sequence of binary bits.
- b. **Source Encoder.** The binary data is first fed into the Source Encoder, whose role is to compress or format the data to reduce redundancy and improve transmission efficiency. The source encoder eliminates any unnecessary bits in the data stream while ensuring that the essential information is preserved. Techniques like Huffman or arithmetic coding can be used here to optimize the data format for transmission.
- c. **Channel Encoder.** After source encoding, the data goes to the Channel Encoder. This encoder, typically a convolutional encoder in TCM systems, adds redundancy to the data, improving its resistance to errors during transmission. The convolutional coding spreads the data across several transmitted symbols, which finds out and correct errors at receiver.
- d. **Modulator.** Encoded data stream is then input to the Modulator, where it is mapped onto a higher-dimensional signal constellation, such as 128-QAM or 256-QAM. In TCM, the modulation is combined with coding to optimize minimum Euclidean distance amongst constellation points, reducing chance of decoding errors. The modulated signal is represented as points in a complex plane, where every point corresponds to unique grouping of input bits.
- e. **Multipath Channel.** The modulated signal is transmitted over the Multipath Channel, which introduces noise, fading, and other distortions. In fading multipath environment, signal may experience constructive or destructive interference, leading to fluctuations in signal strength and potential loss of data integrity.
- f. **Demodulator.** On the receiver side, the signal enters the Demodulator, which maps the received signal back to its corresponding bit sequence. In the presence of noise and fading, the demodulator may not perfectly recover the transmitted symbols, leading to potential errors.
- g. **Channel Decoder.** Subsequently, demodulated data is processed by Channel Decoder, which uses redundancy added during the channel encoding process to detect and correct errors. Typically, the Viterbi algorithm is used here for decoding TCM, as it follows most likely path through trellis structure to reconstruct transmitted sequence.
- h. **Source Decoder.** After error correction, the data stream is passed to the Source Decoder, which reverses any compression or formatting applied by the source encoder. This step reconstructs the data to match the original format of the input source.
- i. **Error Comparison.** Finally, the reconstructed output is compared with the original input to evaluate the error rate. This comparison provides insights into performance of system, highlighting effectiveness of TCM scheme in reducing errors despite noise and fading in the channel.

Through this structured process, the TCM system minimizes errors introduced during transmission, with each block contributing to enhancing data integrity and reliability in challenging channel conditions.

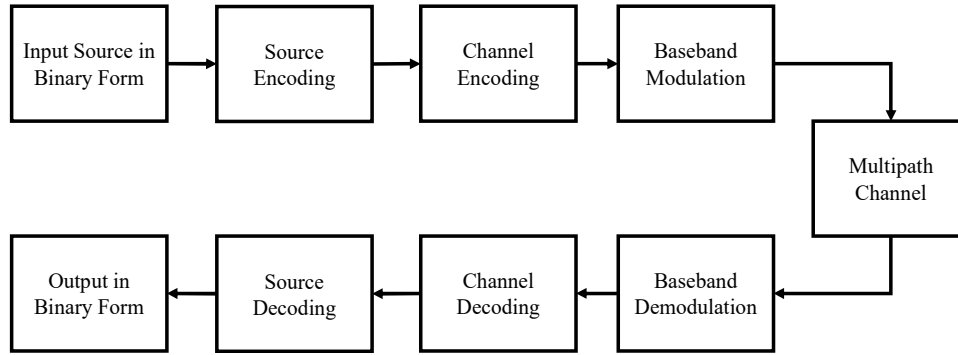


Fig. 2 System block diagram.

Figure 3 presents discrete time model corresponding to scheme depicted in Figure 2. This represents signal received at time index  $i$ , and has been formulated in equation (1)

$$r_i = c_i \cdot s_i + n_i \quad (1)$$

In equation (1), complex white Gaussian Noise with zero mean and variance  $N_0/2$  has been denoted by  $n_i$ . Additionally,  $c_i$  denotes gain of channel which is complex and distribution is Gaussian having  $\sigma^2$  variance.

Alternate equation for  $c_i$  is given in (2).

$$c_i = a_i \cdot e^{j\phi_i} \quad (2)$$

In eq (2), amplitude has been represented by  $a_i$  and phase has been represented by  $\phi_i$ .

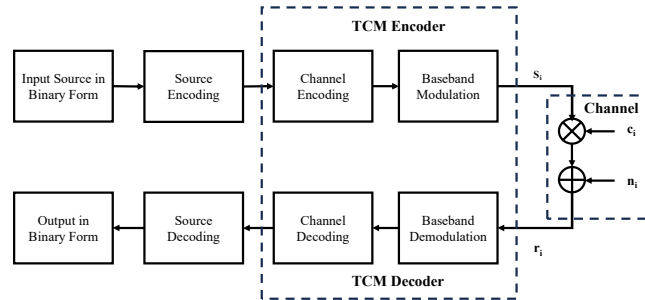


Fig. 3 Baseband representation of System model, shown in Figure 2

Under assumption of coherent detection at receiver, it compensates for phase shift introduced by channel, which enables rewriting equation (1) as follows: -

$$r_i = a_i s_i + n_i \quad (3)$$

In eq (3),  $a_i$  implies noise amplitude.

### III. DESIGN

This section lays the foundation for developing and building rate 7/8, 256-state TCM scheme particularly in respect of channels experiencing fading phenomenon. Design principles serve as a guide for creating TCM scheme that achieve superior performance in context of fading environments. The rules presented here follow similar design criteria earlier introduced for TCM schemes in fading channels with four states and a rate of 2/3 [10,15]. Extending these principles, the design rules for rate 7/8, 256-state TCM codes have been developed to address the specific types of errors introduced by channels experiencing fading phenomenon. Ungerboeck previously employed heuristic methods for designing an 8-state, rate 2 by 3 TCM scheme in respect of fading channels [7], which laid groundwork for further advancements in TCM design.

In this paper detailed guidelines to design 256-state, 256-QAM Trellis Coded Modulation scheme adapted for fading channels has been presented [11]. The key directives are as follows: -

The transitions from one state to other at successive stages are denoted by matrix of  $256 \times 256$ . Each entry in this matrix denotes signal corresponding to transition from  $i^{\text{th}}$  state at  $k^{\text{th}}$  stage to  $j^{\text{th}}$  state at  $(k+1)^{\text{th}}$  stage of trellis.

Matrix captures complete view of signal transitions and structural connections characteristic of TCM scheme, enabling detailed analysis and structuring architecture of system. Each element in the  $i^{\text{th}}$  row of matrix represents signal from paths starting at  $i^{\text{th}}$  state, while elements in  $j^{\text{th}}$  column correspond to signals related to paths arriving at  $j^{\text{th}}$  state. Arrangement transparently represents flow of signals and connections across states in TCM arrangement.

Through application of set partitioning procedure, 256-QAM signal set can be divided in two separate sub-sets:  $A_0 = \{s_0, s_2, s_4, s_6, \dots, s_{254}\}$  and  $A_1 = \{s_1, s_3, s_5, s_7, \dots, s_{255}\}$ . Foundation for these subsets is their least distance between sets, denoted as  $\delta_1$ . By dividing the signal set, it becomes easier to organize and analyze, assisting in identification and application of particular subsets in TCM scheme.

Assignment of signal points to matrix elements follows the rules outlined below: -

- a. According to first rule, every signal must occur no more than once in each designated row or column.
- b. According to second rule, a 256-state, rate 7/8 code does not allow all transitions, as each state has just 128 outgoing paths. Consequently, transition from one state to other can be assigned signal provided LSB of label of originating state corresponds to MSB of label of target state. This condition ensures consistency and alignment within the transition pathways.
- c. All signals corresponding to particular  $y$  value must be drawn solely from either  $A_0$  or  $A_1$  subset. This restriction ensures consistency in signal selection based on the subset associated with the specific  $y$  value.

#### IV. CODE CONSTRUCTION

This section focuses on constructing TCM code for rate 7/8, 256-state, 256-QAM configuration, following the guidelines charted in preceding section. As per second guideline, initial step involves eliminating impermissible changeovers by associating subset  $A_0$  with a Least Significant Bit (LSB) of 0 and subset  $A_1$  with an LSB of 1. Consequently, signals in even-numbered rows will be selected from subset  $A_0$ , while signals in odd-numbered rows will be drawn from subset  $A_1$ .

In this process, any signal point from subset  $A_0$  may be chosen as starting element for first row. Similarly, signal from subset  $A_1$  can be selected as first valid element (representing first permissible transition) for second row in transition matrix. For example, selecting  $s_0$  from subset  $A_0$  and  $s_1$  from subset  $A_1$  as the initial elements of the first and second rows, respectively.

In the following stage, signals from subsets  $A_0$  and  $A_1$  are assigned to the third, fourth, up to the 128<sup>th</sup> row, in alignment with the second guideline.

In the next stage, either  $s_2$  or  $s_6$  may be selected as the initial element of the 129<sup>th</sup> row, with the choice of  $s_3$  or  $s_7$  as initial valid element of 130<sup>th</sup> row. By selecting for  $s_6$  &  $s_7$ , remaining signals are systematically distributed across the remaining rows, following the structure established by the second guideline. This step completes the finalization of the code's design.

#### V. PERFORMANCE ANALYSIS

Performance evaluation is based on a scenario characterized by the channel that is having amplitudes fading independently and does not have memory [14]. Likelihood of occurring error event is of vital importance in evaluating system's performance, as lesser probabilities of error events correspond to fewer errors and thus more reliability. This section is dedicated to deriving probability of error event. Here, we explore the upper bound of pairwise error probability [12]. Assuming coherent detection, ideal CSI, and every symbol has independent fading, following expression represents upper bound of pairwise error probability in symbol sequence deciphering over Rician channel: -

$$P_2(S_l, \hat{S}_l) \leq \prod_{i=1}^l \frac{(1+K)}{1+K+\frac{1}{4N_0}|(S_i-\hat{S}_i)|^2} e^{\left(-\frac{K\frac{1}{4N_0}|(S_i-\hat{S}_i)|^2}{1+K+\frac{1}{4N_0}|(S_i-\hat{S}_i)|^2}\right)} \quad (4)$$

When SNR is large, equation (4) simplifies to:

$$P_2(S_l, \hat{S}_l) \leq \prod_{i \in \eta} \frac{(1+K)e^{-K}}{\frac{1}{4N_0}|(S_i-\hat{S}_i)|^2} \quad (5)$$

Instances where  $S_i$  and  $\hat{S}_i$  differ are captured by  $\eta$ . As a result, eq (5) can be expressed in following form, with the effective error event length  $l_\eta$  referring to  $(S_i, \hat{S}_i)$ .

$$P_2(S_l, \hat{S}_l) \leq \frac{((1+K)e^{-K})^{l_\eta}}{\left(\frac{1}{4N_0}\right)^{l_\eta} d_P^2(l_\eta)} \quad (6)$$

Squared product distance amongst  $S_i$  and  $\hat{S}_i$  along differing points of error event path is represented by  $d_P^2(l_\eta)$ . It is calculated as: -

$$d_P^2(l_\eta) = \prod_{i \in \eta} |(S_i - \hat{S}_i)|^2 \quad (7)$$

where  $\eta$  identifies indices of places  $S_i$  and  $\hat{S}_i$  differ. This product distance is a critical metric in quantifying the impact of errors in high-SNR conditions, emphasizing the cumulative distance over the erroneous signal elements.

Deviation of received data's trajectory from intended path results in an error event, as defined by system's design. Figure 4 provides visual representation of concept, highlighting intended path between  $s_1$  and  $s_2$ , continuing up to  $s_l$ . In contrast, divergent path traced by received symbols is denoted as  $\hat{S}_1, \hat{S}_2 \dots \dots \hat{S}_l$ .

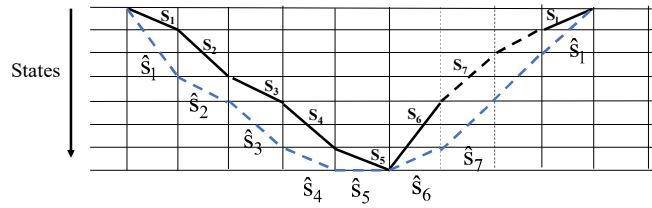


Fig. 4 Error event characterized by length l.

By considering all possible transmitted sequences, bound and upper limit are determined by summing probabilities of error events in respect of all lengths l from 1 to  $\infty$ , as shown below: -

$$P_e \leq \sum_{l=1}^{\infty} \sum_{S_l} \sum_{\hat{S}_l \neq S_l} P(S_l) P_2(S_l, \hat{S}_l) \quad (8)$$

In equation (8),  $P(S_l)$  represents apriori probability of symbol  $S_l$ . At high SNRs,  $P_2(S_l, \hat{S}_l)$  may be replaced in above equation. Subsequently, following expression represents upper bound applicable to Rician channel: -

$$P_e \leq \sum_{l_\eta} \sum_{d_P^2(l_\eta)} \alpha(l_\eta, d_P^2(l_\eta)) \frac{((1+K)e^{-K})^{l_\eta}}{\left(\frac{1}{4N_0}\right)^{l_\eta} d_P^2(l_\eta)} \quad (9)$$

Equation (9) expresses that  $\alpha(l_\eta, d_P^2(l_\eta))$  denotes average count of code sequences having effective length  $l_\eta$  and squared product distance  $d_P^2(l_\eta)$ . While working at large SNRs, error event is influenced by both  $l_\eta$  &  $d_P^2(l_\eta)$ .

For simplification, L is used to denote minimum effective length and  $d_P^2(L)$  to denote squared product distance corresponding to it. Considering this minimal effective length as code's effective length enables approximation of error event probability, expressed below: -

$$P_e \approx \alpha(L, d_P^2(L)) \frac{((1+K)e^{-K})^L}{\left(\frac{1}{4N_0}\right)^L d_P^2(L)} \quad (10)$$

In the Rayleigh fading channel, where the Rician K-factor equals 0 (indicating no line-of-sight component), equation (10) can be further simplified. Since  $K=0$ , the fading distribution becomes purely Rayleigh, and we can express the probability of an error event with this condition as follows: -

$$P_e \approx \frac{\alpha(L, d_p^2(L))}{\left(\frac{1}{4N_0}\right)^L d_p^2(L)} \quad (11)$$

In context of AWGN channel, taking  $K=\infty$  implies that there is dominant line-of-sight component, essentially removing fading effects. Under this assumption, probability of error  $P_e$  can be reformulated as: -

$$P_e \approx \frac{1}{2} N(d_{free}) \operatorname{erfc} \left( \sqrt{\frac{d_{free}^2}{4N_0}} \right) \quad (12)$$

Here,  $d_{free}$  denotes minimum free Euclidean distance, representing smallest Euclidean distance amongst any two distinct code sequences in unrestricted code space. This value  $d_{free}$  is crucial for evaluating the code's error performance, as it sets lower bound on distance that any error event must overcome, thereby influencing the probability of decoding errors.

This section presents performance evaluation of TCM scheme within context of a Rician channel, as referenced in [13, 16]. The analysis primarily focuses on equation (10) with respect to the Rician parameter  $K$ , particularly examining the special cases where  $K=0$  (Rayleigh channel) and  $K=\infty$  (AWGN channel).

The TCM scheme employs convolutional encoders at a coding rate of 7/8. To assess the scheme's effectiveness, results are compared with those of an uncoded 128-QAM modulation. Decoding process makes use of Viterbi Decoding algorithm, optimizing TCM's performance under varying SNR conditions.

Figure 5 provides a visual comparison, plotting BER against SNR in respect of Rate 7/8, 256-State, 256-QAM TCM code developed in accordance with specified guidelines. Additionally, BER versus SNR curve for the uncoded 128-QAM modulation is displayed on the same graph, facilitating a direct comparison between the coded and uncoded schemes. This comparison highlights coding gain attained by TCM scheme over uncoded counterpart, particularly in environments with varying  $K$  values.

## VI. RESULT AND CONCLUSION

The analysis reveals that proposed Trellis Coded Modulation scheme has led to a substantial performance improvement. Specifically, when compared to an uncoded 128-QAM modulation scheme, the TCM design achieves an increase in Signal-to-Noise Ratio (SNR) of over 15 dB. This SNR gain reflects the system's enhanced capacity to resist noise and interference, thereby improving signal clarity during transmission.

Furthermore, the TCM scheme supports higher data transfer rates with elevated reliability, which is especially advantageous over channels susceptible to fading. Fading, caused by factors such as multipath propagation or environmental variations, results in fluctuations in signal strength. By integrating the TCM coding scheme, the communication system becomes significantly more resilient to these fading effects, enhancing overall performance and allowing data transmission at higher rates, even in adverse channel conditions. This robustness marks a key advancement in reliable data communication, underscoring the TCM scheme's suitability for challenging environments.

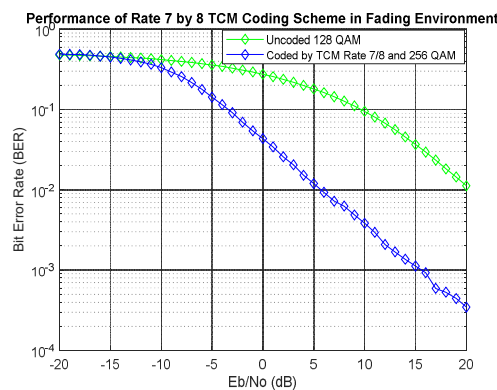


Fig. 5 BER Vs SNR

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